Re- exam Calculus-3 (10 points for free)

Part A: G. Palasantzas (Problems 1-3)

Problem 1
(15 points)

Determine whether the sequence converges or diverges. If it converges, find the limit.

(a: 7 points) $a_n = \dfrac{3 + 5n^2}{n + n^2}$ (b: 8 points) $a_n = \left(1 + \dfrac{2}{n}\right)^n$

Problem 2
(15 points)

Find the radius of convergence and interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}$ with $x \in (-\infty,+\infty)$

Problem 3
(15 points)

consider a spring with mass m, spring constant k, and damping constant $c = 0$, and let $\omega = \sqrt{k/m}$. If an external force $F(t) = F_0 \cos \omega t$ is applied (the applied frequency equals the natural frequency), use the method of undetermined coefficients to show that the motion of the mass is given by

$$x(t) = c_1 \cos \omega t + c_2 \sin \omega t + \frac{F_0}{2m\omega} t \sin \omega t$$

<u>Part B</u>: P.D. Barthel (Problems 4-6)

Problem 4 We consider heat conduction.

(a) A bar with diffusivity 4 and length 6 has its ends kept at zero degrees. Suppose the initial temperature of the bar is 30 degrees. Compute its temperature distribution as a function of time, $u(x,t)$.

(b) Suppose the ends of the bar are insulated and its initial temperature is given by 30 degrees: what will $u(x,t)$ be in this case?

(8 + 7 points).

Problem 5 (a) Show that for $-1 \le x \le 1$:

$$x^2 = \frac{1}{3} + \sum_{n=1}^{\infty} (-1)^n \frac{4}{n^2\pi^2} cos(n\pi x)$$

(b) Show that substitution of a certain value of x yields:

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

(10 + 5 points).

Problem 6 Determine the Fourier Transform $F(\alpha)$ of $f(x)$ defined as $f(x) = \frac{1}{2\epsilon}$ for $|x| < 1$ and $f(x) = 0$ for $|x| > 1$, and subsequently for the same function on the interval $|x| < \epsilon$ and $|x| > \epsilon$. Compute the limit of the last transform for $\epsilon \to 0+$ and discuss the result. (15 points).